Explanatory relations revisited

Links with credibility-limited revision

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2nd Madeira workshop on Belief Revision and Argumentation

Madeira, February 2015

Explanatory relations, a brief introduction

- We study binary relations $\rhd$ over propositional formulas built over a finite set of variables.
- $\alpha \rhd \gamma$ reads α is explained by γ.
- α is the observation and γ is one explanation of α.
- Our goal is to have a better understanding of these abstract relations (behavior, axioms, constructions) and their links with belief revision.

Explanatory relations, a brief introduction (2)

Usually in A.I. [Levesque 89] $\alpha \triangleright \gamma$ imposes that in the light of a theory Σ, γ entails α and $\Sigma \cup \{\gamma\}$ has to be consistent. That is

$\Sigma \cup \{\gamma\} \vdash \alpha$ and $\Sigma \cup \{\gamma\} \nvdash \bot$ (denoted $\gamma \vdash_\Sigma \alpha$)

Actually, γ is in some way one of the "best" formulas in the set $\{\delta : \delta \vdash_\Sigma \alpha\}$.

Different families of explanatory relations and their properties have been studied [Flach 96, 2000, PP-Uzcátegui 99, Bloch et al. 2001]

Explanatory relations, a brief introduction (3)

Some problems:

- Right strengthening:

  $\mathsf{Good\_coffee} \rhd \mathsf{colombian\_coffee} \Longrightarrow$
  $\mathsf{Good\_coffee} \rhd \mathsf{colombian\_coffee\_with\_pepper}$

- Impossible observations:

  $\mathsf{A\_pink\_elephant\_driving\_a\_Fiat\_500}$

- The ground theory Σ:

  Is really necessary to make explicit the ground theory Σ?

Our general approach

Main idea: thinking $\alpha \triangleright \gamma$ as

$\pi_1(\gamma) \vdash \pi_2(\alpha)$

where the functions $\pi_i : \mathcal{L} \longrightarrow \mathcal{L}$ are some sort of "core" functions (giving the more relevant part of the input) satisfying $\pi_i(\beta) \vdash \beta$

Actually we give two families of explanatory relations such that

$\triangleright$ satisfies a set of postulates iff π_1 and π_2 are determined and moreover we have the following representation:

$$\alpha \triangleright \gamma \Leftrightarrow \pi_1(\gamma) \vdash \pi_2(\alpha)$$

Our general approach (2)

More concretely we have the following results:

1. $\rhd$ is an *ordered* explanatory relation iff there are a formula φ and a credibility-limited revision operator $\circ$ such that we have the following representation:

$$\alpha \rhd \gamma \Leftrightarrow \varphi \circ \gamma \vdash \varphi \circ \alpha$$

2. $\rhd$ is an *weakly reflexive* explanatory relation iff there are a formula φ and a credibility-limited revision operator $\circ$ such that we have the following representation:

$$\alpha \rhd \gamma \Leftrightarrow \gamma \vdash \varphi \circ \alpha$$

Our general approach (3)

Some works related to these ideas:

1. Boutilier C., Bescher V., Abduction as belief revision, *Artificial Intelligence* 77 (1995) 43–94.
2. Pino Pérez R., Uzcátegui C., Jumping to explanations versus jumping to conclusions, *Artificial Intelligence* 111 (1999) 131–169.
3. Falappa M. A., Kern-Isberner G., Simari G. R., Explanations, belief revision and defeasible reasoning, *Artificial Intelligence* 143 (2002) 1–28.
4. Walliser B., Zwirn D., Zwirn H., Abductive logics in a belief revision framework, *Journal of Logic, Language and Information* 14 (2005) 87–117.

One example around coffee

Four propositional variables: c, s, p, g meaning Colombian_coffee, with_sugar, with_pepper, good_coffe

We are reasoning about good coffee, so the worlds in which the coffee is no good are impossible worlds. In the same manner the worlds in which there are pepper are incompatible with good coffee. Thus the only credible worlds are in the previous order of the variables $0101, 0001, 1101, 1001$. Suppose the order of these worlds is

$$\begin{array}{ccc} & 0101 & \\ 1101 & & 0001 \\ & 1001 & \end{array}$$

Suppose that π_1 is the identity and π_2 is "taking the minimal models". Then

good_coffee $\rhd$ Colombian_coffee_without_sugar_without_pepper

good_coffee_with_sugar $\rhd$ Colombian_coffee_with_sugar_without_pepper

good_coffee_with_sugar_and_pepper has no explanations!

Recall about Credibility-limited revision

A propositional compact version of [Hansson, S. O., Fermé, E., Cantwell, J. and Falappa, M. Credibility limited revision. Journal of Symbolic Logic, 66:1581–1596, 2001]

Postulates:

$\varphi \circ \alpha \equiv \varphi$ or $\varphi \circ \alpha \vdash \alpha$ (Relative success)

If $\alpha \wedge \varphi \nvdash \bot$ then $\varphi \circ \alpha \equiv \varphi \wedge \alpha$ (Vacuity)

$\varphi \circ \alpha \nvdash \bot$ (Strong coherence)

If $\varphi \equiv \psi$ and $\alpha \equiv \beta$ then $\varphi \circ \alpha \equiv \psi \circ \beta$ (Syntax independence)

If $\varphi \circ \alpha \vdash \alpha$ and $\alpha \vdash \beta$ then $\varphi \circ \beta \vdash \beta$ (Success monotony)

$$\varphi \circ (\alpha \vee \beta) \equiv \begin{cases} \varphi \circ \alpha \text{ or} \\ \varphi \circ \beta \text{ or} \\ (\varphi \circ \alpha) \vee (\varphi \circ \beta) \end{cases} \qquad \text{(Trichotomy)}$$

Recall about Credibility-limited revision (2)

A CL-faithful assignment is a function mapping each consistent formula φ into a pair $(C_\varphi, \leq_\varphi)$ where $[\![\varphi]\!] \subseteq C_\varphi \subseteq \mathcal{V}$, $\leq_\varphi$ is a total total preorder over C_φ, and the following conditions hold for all $\omega, \omega' \in C_\varphi$:

1. If $\omega \models \varphi$, then $\omega \leq_\varphi \omega'$
2. If $\omega \models \varphi$ and $\omega' \not\models \varphi$, then $\omega <_\varphi \omega'$
3. If $\varphi \equiv \varphi'$, then $(C_\varphi, \leq_\varphi) = (C_{\varphi'}, \leq_{\varphi'})$

Recall about Credibility-limited revision (3)

CL Representation [Booth, Fermé, Konieczny and PP 2012]:

Theorem

$\circ$ is a CL revision operator iff there exists a CL-faithful assignment $\varphi \mapsto (C_\varphi, \leq_\varphi)$ such that

$$[\![\varphi \circ \alpha]\!] = \begin{cases} \min([\![\alpha]\!], \leq_\varphi) & \textit{if } [\![\alpha]\!] \cap C_\varphi \neq \emptyset \\ [\![\varphi]\!] & \textit{otherwise} \end{cases}$$

Postulates of ordered explanatory relations

$\rhd \neq \emptyset$	(Non triviality)
$Expl(\alpha) \neq \emptyset \Rightarrow \alpha \rhd \alpha$	(Limited reflexivity)
$\alpha \rhd \gamma \Rightarrow \alpha \wedge \gamma \nvdash \bot$	(Weak infra-classicality)
$\alpha \rhd \gamma, \delta \vdash \gamma, \delta \nvdash \bot \Rightarrow \alpha \rhd \delta$ or $\alpha \wedge \neg\delta \rhd \gamma$	(Weak right strengthening)
$\alpha \rhd \gamma, \gamma \rhd \delta \Rightarrow \alpha \rhd \delta$	(Transitivity)
$\alpha \rhd \gamma, \beta \rhd \gamma \Rightarrow \alpha \wedge \beta \rhd \gamma$	(Left and)
$\alpha \rhd \gamma, \alpha \rhd \delta \Rightarrow \alpha \rhd \gamma \vee \delta$	(Right or)
$\alpha \rhd \gamma, \gamma \vdash \beta \Rightarrow \alpha \wedge \beta \rhd \gamma$	(Cautious monotony)
$\alpha \equiv \alpha', \gamma \equiv \gamma' \Rightarrow (\alpha \rhd \gamma \Leftrightarrow \alpha' \rhd \gamma')$	(Congruence)
$Expl(\alpha) \neq \emptyset, \alpha \vdash \beta \Rightarrow Expl(\beta) \neq \emptyset$	(Explanatory monotony)

Abductive ordered representation

Theorem

$\rhd$ is an ordered explanatory relation iff there exists a consistent formula φ and a credibility-limited revision operator $\circ$ such that

$$\alpha \rhd \gamma \Leftrightarrow (\varphi \circ \gamma \vdash \varphi \circ \alpha), (\varphi \circ \alpha \vdash \alpha) \textit{ and } (\varphi \circ \gamma \vdash \gamma) \qquad (1)$$

Proof key elements:

1. $\varphi \equiv \bigvee \alpha_\omega, \quad \text{where} \quad \alpha_\omega \in \{\alpha_\omega' : \top \rhd \alpha_\omega'\}$
2. $\varphi \circ \alpha \equiv \begin{cases} \varphi & \text{if} \quad Expl(\alpha) = \emptyset \\ \bigvee \alpha_\omega, \ \alpha_\omega \in \{\gamma : \alpha \rhd \gamma\} & \text{if} \quad Expl(\alpha) \neq \emptyset \end{cases}$
3. Establish the representation equivalence (1).
4. Then prove that $\circ$ satisfies the CL revision operator postulates.

Postulates of weakly reflexive explanatory relations

$Expl(\top) \neq \emptyset$	(Strong non triviality)
$\alpha \rhd \gamma \Rightarrow \gamma \nvdash \bot$	(Coherence)
$\alpha \rhd \gamma, \delta \vdash \gamma, \delta \nvdash \bot \Rightarrow \alpha \rhd \delta$	(Right strengthening)
$\alpha \wedge \beta \rhd \delta, \exists \gamma (\alpha \rhd \gamma \text{ and } \gamma \vdash \beta) \Rightarrow \alpha \rhd \delta$	(Weak cut)
$\alpha \rhd \gamma \Rightarrow \gamma \vdash \alpha$	(Infra-classicality)
$\alpha \rhd \gamma, \alpha \rhd \delta \Rightarrow \alpha \rhd \gamma \vee \delta$	(Right or)
$\alpha \rhd \gamma, \gamma \vdash \beta \Rightarrow \alpha \wedge \beta \rhd \gamma$	(Cautious monotony)
$\alpha \equiv \alpha', \gamma \equiv \gamma' \Rightarrow (\alpha \rhd \gamma \Leftrightarrow \alpha' \rhd \gamma')$	(Congruence)
$Expl(\alpha) \neq \emptyset, \alpha \vdash \beta \Rightarrow Expl(\beta) \neq \emptyset$	(Explanatory monotony)

Weak reflexive explanatory relations representation

Theorem

$\triangleright$ is a weak reflexive explanatory relation iff there exists a consistent formula φ and a credibility-limited revision operator $\circ$ such that

$$\alpha \triangleright \gamma \Leftrightarrow (\gamma \vdash \varphi \circ \alpha), (\varphi \circ \alpha \vdash \alpha) \text{ and } \gamma \nvdash \perp \tag{2}$$

Proof key elements:

1. $\varphi \equiv \bigvee \alpha_\omega$, where $\alpha_\omega \in \{\alpha_\omega^{'} : \top \triangleright \alpha_\omega^{'}\}$
2. $\varphi \circ \alpha \equiv \begin{cases} \varphi & \text{if} \quad Expl(\alpha) = \emptyset \\ \bigvee \alpha_\omega,\ \alpha_\omega \in \{\gamma : \alpha \triangleright \gamma\} & \text{if} \quad Expl(\alpha) \neq \emptyset \end{cases}$
3. Establish the representation equivalence (2).
4. Then prove that $\circ$ satisfies the CL revision operator postulates.

Final remarks

1. As a corollary we obtain semantical representations.
2. The theory Σ is actually implicit, it is in fact the theory of $C\varphi$.
3. Now, there are, in general, formulas without explanations.
4. The ordered explanatory relations don't satisfy Right strengthening.
5. The weakly reflexive explanatory relations are in fact an alternative view of the E-rational relations of [PP-Uzcátegui, 1999].
6. To do:
   - Study more schemas (π_1, π_2).
   - Introduce the dynamics (CLIO).